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Surplus Production Models with Auto correlated Errors

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Abstract
In the present investigation, the surplus production models are fitted by incorporating error term with autoregressive of order one for the serially correlated catch-effort dataset from Gobindsagar reservoir. A comparative study among the various models is being possible for fish stock assessment of the above reservoir fisheries. The best model identified is, further, used to estimate the MSY value along with the optimum effort to achieve it.

Keywords: Surplus production models, autocorrelation, fish stock, optimum effort.

1. Introduction
Reservoir resources are important but they are diverse and therefore the strategies to be adopted for optimizing yields are also different [1]. Thus, reservoir fisheries management must be based on an understanding that they are complex and dynamic ecosystem determining an efficient exploitation level of fisheries resources. The fisheries resources cannot sustain if exploitation levels led to biological over fishing as a result of ineffective management. Therefore, rational management is required to rescue the fisheries resources from depletion, to maintain the fisheries productivity capacity dependent on harvesting and many other regimes, so a change of management plan should be achieved by simultaneously changing the level of effort. Furthermore, many of reservoirs have been significantly changed physically and biologically in the last or coupled of decades due to indiscriminate exploitation. In the present scenario, to maintain the population of indigenous fish species and systems for fishery enhancement to a sustainable level, calculation of maximum sustainable yield (MSY) for those reservoirs will be of immense help. Moreover, there is no reliable information on the availability of estimated MSY value for the reservoirs in the literature. Further, the abundance of fish stock in a particular area is a function of interactions between environmental factors and the fish stock properties. The stock tends to stabilize at a particular set of environmental conditions [3]. When the surplus production is not harvested, at the level of maximum fish stock size the addition of recruitment and growth to the stock is just sufficient to compensate for natural mortality and hence, surplus production will equal to zero [4]. This implies that fishing plans can be expressed in terms of surplus production and they are very flexible and have different variations. However, when we are dealing with catch-effort fisheries data, which are observed over continuous time periods, the data points are generally correlated among themselves. Thus, by examining the presence of autocorrelation in the observations, in the present study an attempt has been made to construct surplus production models with serially correlated error structure for accurately estimating the MSY and optimum effort to achieve it.

2. Materials and Methods
2.1 *Surplus Production Models*
A surplus production model facilitates estimation of MSY and the optimum fishing effort for harvesting the MSY (E_{MSY}) [11]. The equilibrium Schaefer model is given by

$$C_t = KE_t\left(1 - E_t/r\right), \qquad \text{-- (1)}$$

where C_t and E_t are catch and effort at time t; r is the intrinsic growth rate; K is the carrying capacity and correspondingly, $MSY = {rK}/{4}$ and $E_{MSY} = {r}/{2}$.

The Fox model [2] is also given below:

$$C_t = KE_t \exp(-{E_t}/{r}), \text{ ----------------------------------- (2)}$$

and its corresponding $MSY = {rK}/{e}$; (e=2.71828) and $E_{MSY} = r$.

Another important surplus production model is due to Pella and Tomlinson [9], called as Pell-Tomlinson model. This model includes both the above models as a special case but at the expanse of an additional parameter (m) and is given by the equation:

$$C_t = KE_t\left(1 - {mE_t}/{r}\right)^{1/m} \text{ ------------------------------- (3)}$$

Here, m is the Pella-Tomlinson shape parameter, and its corresponding

$$MSY = \frac{Kr}{(m+1)^{\left(1+1/m\right)}} \text{ And } E_{MSY} = \frac{r}{(m+1)};\ m \neq -1.$$

As we are dealing with time-series data, it is, therefore, required to check for the validity of the surplus production models by examining the independency assumption of error term. The Durbin-Watson test can be employed for the above purpose and is based on the assumption that the errors (ε_t's) follow autoregressive of order one. The corresponding test statistic'd' is defined as follows:

$$d = \frac{\sum_{t=2}^{n}(\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{n}\varepsilon_t^2},\ 0 \le d \le 4. \text{ -------------------------- (4)}$$

A statistic "d" value ranges between 0 and 4. A value of 'd' near 2 indicates little autocorrelation; a value toward 0 indicates positive autocorrelation while a value toward 4 indicates negative autocorrelation.

To handle a situation when there is an evidence for the presence of autocorrelation, an autoregressive (AR) error term ε_t of order one is added to the right hand sides of above equations:

$$\varepsilon_t = \Phi\varepsilon_{t-1} + u_t;\ |\Phi| < 1, \text{ ------------------------------ (5)}$$

where u_t are independently and normally distributed with zero mean and constant variance and Φ denotes the autoregressive parameter. Incorporating the AR(1) error structure, the above equations (1)-(3) are modified as below:

Schaefer Model with AR(1):

$$C_t = KE_t\left(1 - {E_t}/{r}\right) + \Phi\varepsilon_{t-1} + u_t, \text{ ----------------------- (6)}$$

Fox Model with AR (1):

$$C_t = KE_t \exp(-{E_t}/{r}) + \Phi\varepsilon_{t-1} + u_t, \text{ ------------------- (7)}$$

Pella-Tomlinson Model with AR (1):

$$C_t = KE_t\left(1 - {mE_t}/{r}\right)^{1/m} + \Phi\varepsilon_{t-1} + u_t \text{ --------------- (8)}$$

The unknown parameters along with autoregressive parameter in the above surplus production models and their modified versions are estimated using Levenberg-Marquardt method [12].

2.2 *Measures of Model Adequacy*

This is generally assessed by the coefficient of determination, R^2. However, as pointed out by Kvalseth [8], eight different expressions for R^2 appear in literature. He has emphasized that, although R^2 given by

$$R^2 = 1 - \frac{\sum\left(C_t - \hat{C}_t\right)^2}{\sum\left(C_t - \overline{C}\right)^2};\ t = 1,2,\ldots,n, \text{ ----------------- (9)}$$

is quite appropriate even for nonlinear models, uncritical use of and sole reliance on R^2 statistic may fail to reveal important data characteristics and model inadequacies and hence examination of residuals is strongly recommended. To get more reliability from the results the following statistics in addition to R^2, should also be used:

Root Mean Square Error,

$$RMSE = \left[\frac{\sum\left(C_t - \hat{C}_t\right)^2}{n}\right]^{\frac{1}{2}}$$

Mean Absolute Error,

$$MAE = \frac{\sum\left|C_t - \hat{C}_t\right|}{n}$$

where n is the number of observations; $\hat{C}_t$ is the predicted fish catch at time t.

The lower the value of these statistics, the better is the fitted model. It is, further, recommended for residual analysis to check the assumptions made for the model to be developed. Thus, independence or the randomness assumption of the residuals needs to be tested before taking any final decision about the adequacy of the model developed. To test the independence assumption of residuals Run test procedure is available in the literature [10]. However, the normality assumption is not so stringent for selecting non-linear models because their residuals may not follow normal distribution.

3. Illustration

Gobindsagar reservoir is situated at a distance of 105 km to the northeast of Chandigarh at 31^0 25' N latitude and 76^0 25' E longitude. Gobindsagar is 90 kms long and encompasses an area of approximately 170 sq kms. The water-spread area varies from a minimum of 60 sq km at DSL (dead storage level) of 445.62 m to 168.35 sq km at FRL (full reservoir level) of 513.98 m. The water surface area is found maximum during September and minimum during May. The reservoir has a maximum depth of 163 m and a mean depth of 55 m. The shore development of 12.86 indicated highly irregular shorelines of the reservoir. Fishing is traditionally an important occupation for local people living in reservoir areas and has a main role as a protein source in the diets of many households. Thus, fishing is a regular activity and 51 species and sub-species have been recorded. Evaluation of limnological parameters and tropic status for formulation for suitable management technique was monitored for a number of years for this reservoir. The important studies on these aspects of Gobindsagar had been done [5, 6]. Also, investigations were conducted on many aspects and data on fishing effort and its corresponding yield or catch in Gobindsagar reservoir during 1974-75 to 1989-90 were observed [7]. Although aquatic resources in the reservoir are highly diversified in species composition as well as abundant primary productivity, these resources have been reduced in terms of quantity and size of fish caught in recent years. It is, therefore, a better fishery management needs to be imposed to maintain productivity of the fisheries resources on a sustainable basis. The studies so far reported on Gobindsagar reservoir showed that less attention has been paid to sustainable use of fish resources. Therefore, in the present investigation, the most popularly known surplus production models will be fitted to the data obtained [7] to estimate the approximate value of MSY and optimum effort of Gobindsagar reservoir which will contribute to fill some of the existing gap of studies on the analysis of sustainable fishery of this reservoir.

4. Results and Discussion

In order to fit the surplus production models to the available dataset, the Levenberg-Marquardt method for fitting of non-linear models is used. The main purpose is to obtain estimates of MSY and the corresponding optimum effort to achieve it. The MSY and its corresponding optimum efforts of four different surplus production models are computed since the Pella-Tomlinson model fails to give optimal solution and the results of the fitted models are presented in Table (1). Schafer model provides a curve of symmetrical parabola as shown in Fig. (1), which need not necessarily be always true to the natural phenomena since MSY may vary considerably depending on temporal and spatial changes in the ecosystem. However, catch increases asymmetrically towards the total biomass with increasing effort in Fox model as shown in Fig. (2), which is more realistic than Schaefer model. The MSY values (in tones) estimated by Schaefer and Fox models are 808 and 974; and their corresponding optimum efforts (no. of gill nets) are 1406, and 2252 respectively. Although normality assumption regarding the error term in catch are met for both the models since Shapiro-Wilk test p-values of the fitted models are 0.123 and 0.125 respectively, however randomness assumption does not follow since the run test $|Z|$ value (2.303) is greater than 1.96 of normal distribution at 5% level of significance. Further, Durbin-Watson statistics have been calculated and the statistic values are toward zero and hence positive autocorrelation is suspected. The Schaefer and Fox models are refitted incorporating the AR(1) error structure and the corresponding results are again shown in Table (1). The graphs of fitted Schafer and Fox models with AR(1) along with observed catch values are shown in Fig. (3). Further, run test and Shapiro-Wilk test are employed to the residuals of catch. Here, the run test $|Z|$ value and Shapiro-Wilk test p-values of the refitted models indicate that both the randomness and normality assumptions are satisfied. The Durbin-Watson statistic values calculated from the refitted models are also very closed to 2 i.e. presence of autocorrelation is negligible. Also, a significant improvement in R^2, RMSE and MAE values is seen in the refitted models as compare to the original Schaefer and Fox models. However, Schaefer model with AR(1) shows better performance to its counterpart of Fox model when the criteria of R^2, RMSE and MAE are used to decide the best fitted model to the data. A perusal of the estimates of MSY for different surplus production models reveals that the four estimates do not differ distinctly but it seems that Schaefer and Fox models other than those models with AR(1) error structure over-estimates the MSY value. The above fish catch data from Gobindsagar reservoir has also yielded an average annual catch of 586 tones, corresponding to an average annual effort of 680 (no. of gill nets) which are very close to the respective values of 637 (tones) and 879 (no. of gill nets) estimated by Schaefer model with AR(1) rather than other models.

Table 1: Summary statistics for fitting of various surplus production models on data collected from Gobindsagar reservoir

	Schaefer Model	**Schaefer Model with AR (1)**	**Fox Model**	**Fox Model with AR (1)**
Parameters				
K	1.149	1.449	1.176	1.544
r	2811.287	1758.330	2252.098	1216.731
Φ	-	0.630	-	0.614
Statistics				
MSY (in tones)	808	637	974	691
E_{MSY} (no. of gill nets)	1406	879	2252	1217
Durbin-Watson statistic	0.843	1.753	0.847	1.735
Model adequacy				
R^2	0.238	0.493	0.237	0.486
RMSE	140.292	114.455	140.412	115.264
MAE	123.802	96.767	123.696	97.795
Residual analysis				
Run test $(\lvert Z \rvert)$	2.303	0.259	2.303	0.259
Shapiro Wilk test p-value	0.123	0.231	0.125	0.231

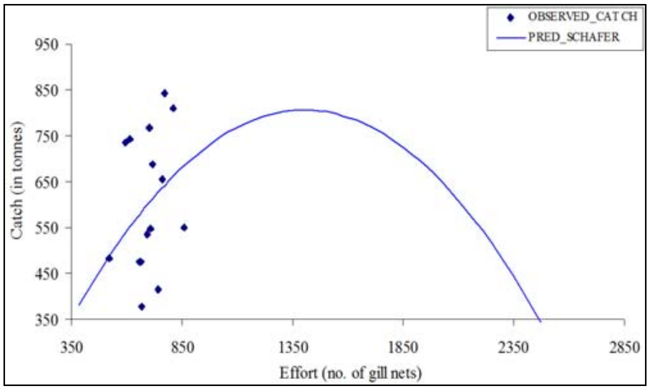

Fig 1: Fitted equilibrium Schaefer model

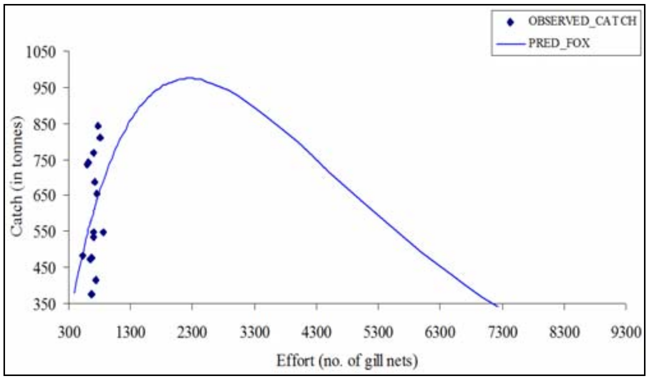

Fig 2: Fitted equilibrium Fox model

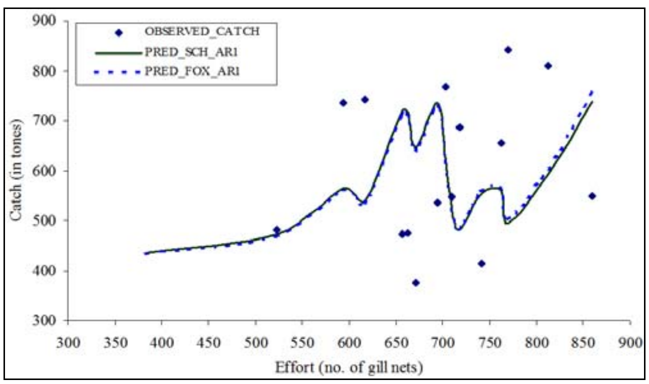

Fig 3: Comparison of catch-effort curves of different surplus production models with autocorrelated errors

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